Eoundation for success
UIM
Unified International
Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)



KEY

| 1 | 2 | 3 | 4 | 5 | $\mathbf{6}$ | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | D | A | B | C | D | C | B | B | D |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| B | A | C | Delete | C | C | A | D | B | C |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| B | C | B | A | C | D | B | A | C | D |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| $\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}$ | $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ | $\mathrm{A}, \mathrm{B}, \mathrm{D}$ | $\mathrm{A}, \mathrm{B}, \mathrm{C}$ | $\mathrm{B}, \mathrm{C}$ | B | A | B | D | C |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| C | B | C | C | A | C | A | B | C | C |

## EXPLANATIONS

## MATHEMATICS - 1

1. (C) $1.32 y+0.02 y-y=0.595$
$0.34 y=0.595$
$y=\frac{0.595}{0.34}=\frac{59.5^{3.5}}{34_{2}}=\frac{3.5}{2} \times \frac{2}{2}=\frac{7}{4}$
2. (D) Let the number of people be ' $x$ '
$\therefore \quad$ Each person contribution $=₹ x$
$\therefore \quad$ Total amount $=(x)(x)=₹ x^{2}$
Given ₹ $x^{2}=$ ₹ 2209
$x^{x}=(47)^{x}$
Number of persons in the group $(x)=47$
3. (A) Given solid is a triangular prism
4. (B) Volume of the box $=(2.6 \mathrm{~m})^{3}=17.576 \mathrm{~m}^{3}$
5. (C) $\left[\frac{1}{x-2}-\frac{4}{x^{2}-4}\right]=\frac{x+2-4}{\left(x^{2}-4\right)}$
$=\frac{(x-2)^{1}}{(x-2)_{(x+2)}}=\frac{1}{(x+2)}$
6. (D) Required difference
$=₹ 1000\left(1+\frac{1 \varnothing}{10 \varnothing}\right)^{4}$
$-₹ 1000-\frac{₹ 1000 \times 4 \times 10}{100}$
$=₹ 1000 \times \frac{14641}{10,000}-₹ 1000-₹ 400$
= $1464.1-1400$
= ₹ 64.1
7. (C) Excluded number $=29 \times 5-27 \times 4$
$=145-108$
$=37$
8. (B) $\left(x^{2}+y^{2}-x y\right)\left(x^{3}+y^{2}+x y\right)$
$=\left(x^{2}+y^{2}\right)^{2}-(x y)^{2}$
$=x^{4}+2 x^{2} y^{2}+y^{4}-x^{2} y^{2}$
$=x^{4}+x^{2} y^{2}+y^{4}$
9. (B) Cost of each piece $=\frac{₹ 112.50}{3}=₹ 37.50$
$\therefore$ Number of pieces for
$₹ 487.50=\frac{₹ 487.50}{₹ 37.5}=13$
10. (D) Given $\frac{P \times \nsim 2 \times r}{100_{50}}=₹ 400$
$\therefore \quad \operatorname{Pr}=50 \times ₹ 400=₹ 20000$
Given
$P\left(1+\frac{r}{100}\right)^{2}-P-\frac{P \times r \times 2}{100}=₹ 410-₹ 400$
$\Rightarrow P\left(1+\frac{2 r}{100}+\frac{r^{2}}{10000}\right)-P-\frac{2 P r}{100}=₹ 10$
$\Rightarrow \not p+\frac{2 P \not{ }^{\prime}}{100}+\frac{P^{2}}{10000}-\not p-\frac{2 P \not p}{100}=₹ 10$
$\therefore \frac{\operatorname{Pr} \times r}{10000}=₹ 10$

$$
\frac{20,000^{2} \times r}{10,000}=10
$$

$\therefore r=\frac{10}{2}=5 \%$
11. (B) Given $2 \sqrt{x}=2+\sqrt{12}=2+2 \sqrt{3}$

Cubing on both sides
$\sqrt{x}=\frac{2(1+\sqrt{3}}{2}$
Squaring on both sides

$$
\begin{aligned}
& x=(1+\sqrt{3})^{2} \\
& =1+3+2 \sqrt{3} \\
& =4+2 \sqrt{3}
\end{aligned}
$$

12. (A) $x^{2}-y^{2}=(\sqrt{2})^{2}-\left(\frac{1}{\sqrt{2}}\right)^{2}$
$=2-\frac{1}{2}=\frac{4-1}{2}=\frac{3}{2}$
13. (C) Speed per minute $\left(S_{1}\right)=\frac{37 \mathrm{~km}}{50}$
$\therefore \quad$ Total distance $=$ Distance travelled for 6 hours

$$
\begin{aligned}
& =\frac{37 \mathrm{~km}}{50 \mathrm{~min}} \times 360 \mathrm{~min} \\
& =\frac{1332}{5} \mathrm{~km}
\end{aligned}
$$

Speed per minute
$\left(S_{2}\right)=\frac{60 \mathrm{~km}}{1 \frac{1}{4} h}=\frac{60 \mathrm{~km}}{\frac{5}{4} h}$
$=\frac{48^{4} \mathrm{~km}}{6 \sigma_{5} \mathrm{~min}}$
$\therefore \quad$ Time taken to cover $\frac{1332 \mathrm{~km}}{5}=\frac{\mathrm{d}}{\mathrm{S}_{2}}$
$=\frac{\left(\frac{1332}{5} \mathrm{~km}\right)}{\left(\frac{4 \mathrm{~km}}{5 \mathrm{~min}}\right)}$
$=\frac{1332^{333}}{\not D} \times \frac{\not D}{\not A} \min$
$=5$ hours 33 minutes
14. (Delete)
15. (C) Given $y>x \Rightarrow \frac{1}{y}<\frac{1}{x}$
16. (C)


In $\triangle \mathrm{ABP}, 45^{\circ}+45^{\circ}+\angle \mathrm{P}=180^{\circ}$
$\Rightarrow \angle \mathrm{P}=90^{\circ}$
Similarly $\angle \mathrm{Q}=\angle \mathrm{R}=\angle \mathrm{S}=90^{\circ}$
In a quadrilateral $P Q R S, P S=P Q$
$\therefore \quad$ PQRS is a square
17. (A) 5 units difference $=15$ cell phone
$\therefore 1$ unit $=\frac{15^{3}}{5}=3$
$\therefore \quad$ No. of cell phones sold on Monday $=7 \times$
$3=21$
18. (D) Given $A: B=2: 3 \& B: C=5: 8$
$\therefore \quad A: B: C=2 \times 5: 3 \times 5: 8 \times 3=10: 15: 24$
$=10 x: 15 x: 24 x$
Given $10 x+15 x+24 x=98$
$49 x=98$
$x=\frac{98}{49}=2$
$C=24 x=24 \times 2=48$
19. (B) Decreased percentage $=\frac{70-49}{70} \times 100$
$=\frac{21^{3}}{70_{1 \varnothing}} \times 10 \varnothing=30 \%$
20. (C) Area of rectangle $=l \times b$

$$
\begin{align*}
& =\left(x^{3}+x^{2} y+x y^{2}+y^{3}\right)(x-y) \mathrm{cm}^{2} \\
& =x^{3}(x-y)+x^{2} y(x-y)+x y^{2}(x-y)+y^{3}(x-y) \\
& =x^{4}-x^{3} y+x^{3} y-x^{2} y^{2} \\
& +x^{2} y^{2}-x y^{5}+x y^{5}-y^{4} \\
& =\left(x^{4}-y^{4}\right) \mathrm{cm}^{2} \tag{1}
\end{align*}
$$

21. (B) Given $3^{x-y}=3^{3} \Rightarrow x-y=3 \rightarrow$

Given $3^{x+y}=243=3^{5}$
$\Rightarrow x+y=5 \rightarrow(2)$
$\mathrm{eg}(2)-(1)(x+y)-(x-y)=5-3$
$\not x+y-\not x+y=2$
$2 y=2$
$y=\frac{\mathfrak{Z}^{1}}{\underline{2}}=1$
22. (C) $6 p^{2}+p-12$

$$
\begin{aligned}
& =6 p 2+9 p-8 p-12 \\
& =3 p(2 p+3)-4(2 p+3) \\
& =(3 p-4)(2 p+3)
\end{aligned}
$$

23. (B) Length of rectangle

$$
\begin{aligned}
& =\frac{\text { Area }}{\text { breadth }}=\frac{5 \mathrm{~cm}^{2}}{\left(\frac{20}{13}\right) \mathrm{cm}} \\
& =\$ 8 \mathrm{~cm}^{2} \times \frac{13}{20_{4} \mathrm{~cm}} \\
& =\frac{13}{4} \mathrm{~cm} \\
& =3 \frac{1}{4} \mathrm{~cm}
\end{aligned}
$$

24. (A) $4^{2}+7.5^{2}=16+56.25=72.25=(8.5)^{2}$ $\therefore 4,7.5 \& 8.5$ are pythagorean triplets
25. (C) Except option ' $C$ ' remaining are rational numbers
26. (D) Given $(2 x-1) \times 4 x \mathrm{~cm}^{2}=60^{15} \mathrm{~cm}^{2}$
$2 x^{2}-x-15=0$
$2 x^{2}-6 x+5 x-15=0$
$2 x(x-3)+5(x-3)=0$
$(x-3)(2 x+5)=0$
$x-3=0 \quad$ (or) $\quad 2 x+5=0$
$x=3$

$$
x=-\frac{5}{2}
$$

27. (B) Given $l=3 \times 4 \mathrm{~cm}=12 \mathrm{~cm}, \mathrm{~b}=3 \mathrm{~cm} \& \mathrm{~h}$
$=3 \mathrm{~cm}$
$\therefore$ Total surface area $=2(l b+b h+h l)$
$=2(12 \times 3+3 \times 3+3 \times 12) \mathrm{cm}^{2}$
$=2 \times 81 \mathrm{~cm}^{2}$
$=162 \mathrm{~cm}^{2}$
28. (A) Let the number of days be ' $x$ ' No. of students Charge No.days

| 35 | $₹ 6300$ | 24 |
| :--- | :--- | :--- |
| 25 | $₹ 3375$ | $x$ |

No. of students \& no. of days are inversely proportional and charge is directly proportional to no. of days
$\therefore 25^{5} \times 6300^{900^{36^{4}}}: 35^{7} \times 3375^{1335^{25^{3}}}=24: x$
$\therefore 4: 3=24: x$
$4 x=24 \times 3 \Rightarrow x=\frac{72}{4}=18$ days
29. (C) Let the number be ' $x$ '

Given $x\left(1+\frac{15^{3}}{100_{20}}\right)=184$
$x\left(\frac{20+3}{20}\right)=184$
$x=184^{8} \times \frac{20}{23_{1}}=160$
$\therefore$ Required number $(x)=160$
30. (D) Given $\sqrt{1+\frac{25}{144}}=1+\frac{x}{12}$
$\sqrt{\frac{144+25}{144}}=1+\frac{x}{12}$
$\sqrt{\frac{169}{144}}=1+\frac{x}{12}$
$\frac{13}{12}=1+\frac{x}{12}$
$\not \lambda+\frac{1}{12}=\nsim+\frac{x}{12}$
$\therefore x=1$

## MATHEMATICS - 2

31. (A, B, C, D)

Options A, B, C \& D are true
32. (A, B, C, D)
$17^{2}-13^{2}=289-169=120$
$\therefore 120+1=121=11^{2}$
$120+49=169=13^{2}$
$120+76=196=14^{2}$
$120+24=144=12^{2}$
33. (A, B, D)

Except ' $C$ ' remaining options are rational numbers $\frac{-62}{0}$ is not defined which is not a rational number
34. (A, B, C)

Given $x \alpha \frac{1}{y} \Rightarrow x_{1} y_{1}=x_{2} y_{2}=x_{3} y_{3}$
$\therefore 10 \times 6=15 \times 4=300 \times 0.2=1200 \times 0.05$
35. (B, C) Only B \& C options are true

## REASONING

36. (B) Pattern of the series as shown below.

37. (A) Option A is the correct answer as the order in which the girls are sitting is Radha, Revati, Janhvi, Harshada, Nellima.
38. (B) May $20^{\text {th }}, 2020$ is wednesday May + June + July + Aug + Sept + Oct + Nov $11+30+31+31+30+31+3=167 / 7=23$ 6 odd days from wed, tuesday is the $3^{\text {rd }}$ nov, 2020
39. (D)

40. (C) Overlaping first two images we get third image.

41. (C) In these type of questions, a common mistake is to miss or double count a particular side or corner. The best way to avoid this mistake is to keep marking the side/corner which has been counted.

42. (B) In any cube, for any of its sides, 4 sides will always be adjacent and one side will always be opposite to it. For example in the green coloured side of the cube below, only the side coloured red is opposite to it and all others are adjacent to it.

If all numbers adjacent to 5 are less then 5 , then $1,2,3$ and 4 are adjacent to 5 . So 5 and 6 are opposite faces.

43. (C) ' G ' is coded as ' $\because$ '
' $R$ ' is coded as '*'
' A ' is coded as ' $@$ '
' $P$ ' is coded as '?'
' $E$ ' is coded as ' $x$ '
' S ' is coded as ' $\%$ '
44. (C)

45. (A) Smallest triangles $=\triangle A O D, \triangle A O E, \triangle C O E$, $\triangle B O D, \triangle B O C=5$

Triangles formed with two triangles
$=\triangle \mathrm{AOC}(\triangle \mathrm{AOE}+\triangle \mathrm{COE}), \triangle \mathrm{AOB}(\triangle \mathrm{AOD}+$ $\triangle \mathrm{BOD}), \mathrm{BCD}(\triangle \mathrm{BOD}+\triangle \mathrm{BOC}), \triangle \mathrm{BCE}$ $(\triangle \mathrm{BOC}+\triangle \mathrm{COE})=4$

Triangles formed with three triangles
$=\triangle \mathrm{ACD}(\triangle \mathrm{AOD}+\triangle \mathrm{AOE}+\triangle \mathrm{COE}), \triangle \mathrm{ABE}$ $(\triangle \mathrm{BOD}+\triangle \mathrm{AOD}+\triangle \mathrm{AOE})=2$
Largest triangle $=\triangle \mathrm{ABC}=1$
Total triangles $=5+4+2+1=12$


## CRITICAL THINKING

46. (C)

47. (A) Statement $1: 9^{\text {th }}$ floor have wall to wall carpeting.

Statement 2 : No wall to wall carpeting is pink.

So, None of offices on the $9^{\text {th }}$ floor has pink wall to wall carpeting.

Hence, $3^{\text {rd }}$ statement is true.
48. (B) Chaitan is heavier than Charan
(A) Kiran is as heavy as Lalith (partially true)
(B) Chaitan is heavier than Charan (true)
(C) Kiran is not heavier than Lalith (partially true)
(D) Kiran is as heavy as Chaitan and Lalith (false)
49. (C)

50. (C) C

